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Mathematical Modelling on unsteady oscillatory flow of blood in an inclined tapered stenosed artery with permeable wall

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Abstract- The purpose of this work is to study the Mathematical modeling on unsteady MHD Oscillatory blood flow in an inclined tapered stenosed artery with permeable wall with effects of slip velocity. The governing equation are solved by analytically by using Bessel function. The effects of slip velocity with different parameters on axial velocity profile, volumetric flow rate and wall shear stress of the oscillatory are solved analytically. Some of the found results show that the flow patterns with slip effects in non-tapered region $(\xi = 0)$ and diverging region $(\xi > 0)$ are effectively influenced by the presence of inclined magnetic field and change in leaning of artery, with time varying pressure gradient. Results for the effect of permeability on these flow characteristics are shown graphically and discussed briefly.

1. INTRODUCTION

The investigations of blood flow through inclined tapered artery is considerable importance in many cardiovascular diseases particularly atherosclerosis (medically called stenosis). In developed and developing countries, one of the major health hazards is atherosclerosis, which refers to the narrowing of the arterial lumen. The main disadvantage is using a tapered geometry however, is the much greater energy losses which may leads to diminished blood flow through the tapered grafts. It is important therefore the losses are quantified and taken account in the design of tapered grafts. The arterial MHD pulsatile flow of blood under periodic body acceleration has been studied by Das et al., (2009). A mathematical model for blood flow through inclined stenosed artery is studied by Siddiqui and Geeta (2016). Mathematical modeling of blood flow in an inclined tapered artery under MHD effect through porous medium was analyzed by Ajaykumar et al., (2016). Slip effects on the Unsteady MHD pulsatile blood flow through porous medium in an artery under the effect of body acceleration is studied by Islam (2012). A numerical analysis for the effect of slip velocity and stenosis shaper on non-Newtonian flow of blood is worked by Bhatnagar et al., (2015). In view of the above mentioned fact, we are analyzing the characteristics of the blood flow of an inclined tapered artery with mild stenosis under the influence of an inclined magnetic field and time varying pressure gradient through porous medium. The analysis is carried out by employing analytically solutions and some important predictions have been made basing upon the unsteady. This unsteady with slip effect can play a big role in the conclusion of axial velocity, wall shear stress and volume flow rate through porous medium. This study is also useful for effects the role of porosity.

2. FORMULATION OF THE PROBLEM

The geometry of an arterial non-symmetrical stenosis in a tapering wall can be expressed (Mekheiner and Kothari, 2008) as

$$R(z) = \begin{cases} h(z)[1-\eta(b^{n-1}(z-a)-(z-a)^2)]; a \le z \le a+b \\ h(z) \quad ; \text{other wise} \end{cases} \tag{1}$$

with $$h(z) = h_0 + \xi z \tag{2}$$

where $R(z)$ is the radius of the stenosed portion of arterial segment and $h(z)$ radius of tapered arterial segment in the stenotic region, h_0 is the radius of the non tapered artery in the non stenotic region, ξ is the tapering parameter, b is taking the length of the stenosis, and $(n \ge 2)$ is being a parameter determining the shape constriction and referred to as a shape parameter.

Here we are using the parameter η which is given by:

$$\eta = \frac{\delta}{h_0 b^n}\left(\frac{n^{\frac{1}{(n-1)}}}{n-1}\right) \tag{3}$$

where δ denotes the maximum height of the stenosis to be found at :

$$z = a + \frac{b}{n^{\frac{1}{(n-1)}}} \tag{4}$$

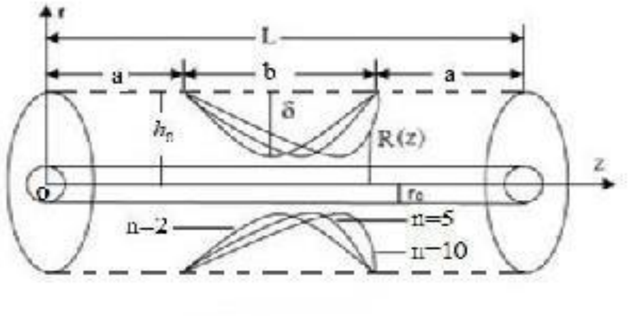


Fig. 1 : Geometry of construction

Here the body fluid is assumed to behave as a Newtonian fluid (Schlitchting and Gerstein 2004) . The equation (as obtained from Navier-Stokes equation of motion for various fluids) describing the unsteady flow of Newtonian fluid is given by (Schlitchting and Gerstein 2004):

$$\frac{\partial p}{\partial r} = 0 \tag{5}$$

$$\frac{\partial p}{\partial \theta} = 0 \tag{6}$$

As per the published literature and available physiological data, blood flow in the neighborhood of the vessel wall can be considered as Newtonian, if the shear rate of blood is high enough. However, the shear rate is very small towards the center of the artery (circular tube), the non-Newtonian behaviour of blood is more evident (Mishra et al., 2007).

The unsteady flow of blood through the cylindrical artery inclined at an angle α can write as follows:

$$\nabla.\bar{u} = 0 \tag{7}$$

$$\frac{\partial \bar{u}}{\partial t} + \rho \bar{F}\bar{u} = -\nabla p + \mu\left[\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial \bar{u}}{\partial \bar{r}}\right] + \bar{J}X\bar{B} \tag{8}$$

Boundary conditions for the problem [Beavers and Joseph (1967)]

$$\left.\begin{aligned} &\textbf{Slip condition} &&: u = u_B \text{ and } \frac{\partial u}{\partial r} = \frac{\gamma}{\sqrt{D_a}}\left(u_B - u_p\right) \text{ at } r = R(z) \\ &\textbf{Symmetry condition :} && \frac{\partial u}{\partial r} = 0 \text{ at } r = 0 \end{aligned}\right\} \tag{9}$$

where $u_p = \frac{-D_a}{\mu}\frac{dp}{dz}$, u_p is the velocity in the permeable boundary, u_B is the slip velocity, D_a is the Darcy number and γ is the slip parameter, a dimensionless quantity depending on the material parameters which characterize the structure at the permeable material within the boundary region.

3. MATHEMATICAL SOLUTION

The equation (8) transformed in the term as:

$$\frac{\partial \bar{u}}{\partial t} + \rho g \sin \alpha \bar{u} = -\frac{\partial \bar{p}}{\partial \bar{z}} + \mu \left[\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}}{\partial \bar{r}} \right] - \sigma B_0^2 \bar{u} \tag{10}$$

Where $\bar{J} X \bar{B} = -\sigma B_0^2 \bar{u}$, g is the acceleration due the gravity, μ is the viscosity of blood, ρ is the fluid density, α is the inclination of an artery, B_0 is an applied magnetic field with an inclination θ.

The non-dimensional variables are :

$$r = \frac{\bar{r}}{R_0}, z = \frac{\bar{z}}{R}, R = \frac{\bar{R}}{R_0}, u = \frac{\bar{u}}{u_0}, p = \frac{\bar{p} R_0^2}{b \mu u_0}, \mathrm{R}_e = \frac{\rho u_0 R_0}{\mu}, F_r = \frac{u_0^2}{g R_0}, M^2 = \frac{\sigma R_0^2 B_0^2}{\mu} \tag{11}$$

Where $\bar{u}$ is velocity component in the axial $\bar{z}$ and radial $\bar{r}$ directions, $\bar{p}$ the pressure, ρ is the density, R_0 is the radius of the normal artery δ is the maximum height of the stenosis, R_e is the Reynolds number, F_r is the Froude number, M is the Hartmann number.

Substituting (11) in (10), we can get a dimensionless as :

$$\frac{\partial u}{\partial t}\left(\frac{R_0^2}{\mu}\right) = -\frac{\partial \bar{p}}{\partial \bar{z}}\left(\frac{b}{R(z) R_0}\right) + \left[\frac{\partial^2 \bar{u}}{\partial \bar{r}^2} + \frac{1}{\bar{r}} \frac{\partial \bar{u}}{\partial \bar{r}}\right] - \left[\frac{\mathrm{R}_e}{F_r} \sin \alpha + M^2 \cos \theta\right] u \tag{12}$$

As the flow is unsteady and axi-symmetric, let the solution for u(r,t) and p be set in the forms :

$$u(r,t) = u(r) e^{i\omega t} \text{ and } -\frac{\partial p}{\partial z} = P(z) e^{i\omega t} \tag{13}$$

Were P is a constant. Substitute equation (13) in (12) we can have a second order ordinary differential equations as follows :

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \beta^2 u = -C \tag{14}$$

where $\beta^2 = \frac{\mathrm{R}_e}{F_r} \sin \alpha + M^2 \cos \theta - \frac{i \omega R_0^2}{\mu}$ and $C = -P(z)$.

The solution of second order differential equation (14) by using boundary conditions, we get

$$u(r,t) = \left(\frac{C}{\beta^2} + \frac{J_0(i\beta.r)}{J_0(i\beta.R(z))} \left(u_B - \frac{C}{\beta^2} \right) \right) \tag{15}$$

then u_B is the slip velocity is determine a

$$u_B = \frac{1}{\frac{\alpha}{\sqrt{D_a}} + \frac{J_1(i\beta.R(z)) i\beta}{J_0(i\beta.R(z))}} \left(\frac{\alpha}{\mu} \sqrt{D_a} - \frac{J_1(i\beta.R(z)) i}{J_0(i\beta.R(z)) \beta} \right) C$$

Substitute the value u(r) in equation (15) we get,

$$u(r,t) = \left(\frac{C}{\beta^2} + \frac{J_0(i\beta.r)}{J_0(i\beta.R(z))} \left(u_B - \frac{C}{\beta^2} \right) \right) e^{i\omega t} \tag{16}$$

Where J_0 and J_1 are the modified Bessel's function of the zero order.

The Volumetric flow rate Q of the blood flow of stenotic analysis

$$Q = 2\pi R_0^2 \int_0^{R(z)} r u dr \quad Q = 2\pi R_0^2 \left[\frac{R^2(z).C}{2\beta^2} + \frac{R(z)}{i\beta} \frac{J_1(i\beta.R(z))}{J_0(i\beta.R(z))} \left(u_B - \frac{C}{\beta^2} \right) \right] e^{i\omega t} \qquad (17)$$

The Wall shear stress is defined by,

$$\tau_w = \left[-\mu \frac{\partial u}{\partial r} \right]_{r=R(z)} \qquad \tau_w = -\left(\frac{J_1(i\beta.R(z)).i\beta}{J_0(i\beta.R(z))} \left(u_B - \frac{C}{\beta^2} \right) \right) e^{i\omega t} \qquad (18)$$

4. RESULTS AND DISCUSSION

Most of the theoretical result such as the effects of various parameters Froude number (F_r), inclination angle of artery (α), Slip parameter (γ), time (t), Hartmann number (M), the inclination of magnetic field (θ) on the flow characteristics wall shear stress(**τ**), axial velocity (u) and volumetic flow rate (Q) are computed graphically

All graphs are plotted by using mathematical software MATLAB for the fixed value R_e=0.1, F_r=0.1, M=2,3 and 4, t=0.1,0.2 and 0.3, δ=0.10, 0.15, 0.20.

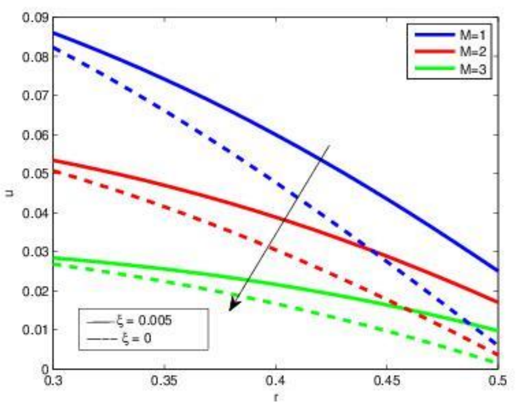


Fig.2 : Variation of axial velocity u with radius r for

for different values of Magnetic field M.

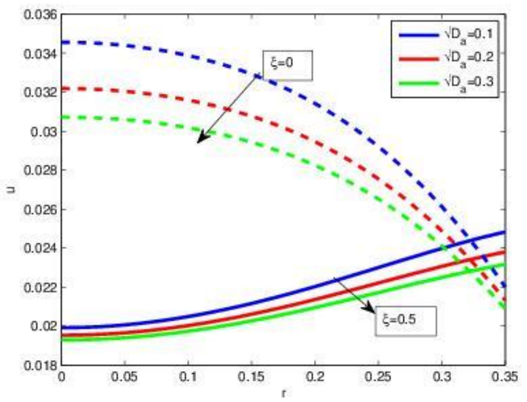


Fig.3 : Variation of axial velocity u with radius r

different values of Darcy number $\sqrt{D_a}$.

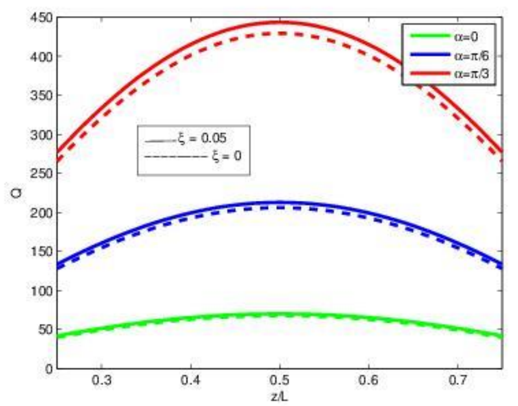
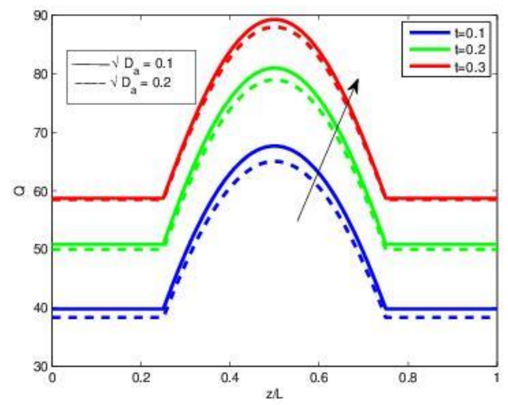


Fig.4 : Variation of volumetric flow rate Q with z/L z/L



F ig. 5 Variation of volumetric flow rate Q with for different values of angle of artery inclination (α).

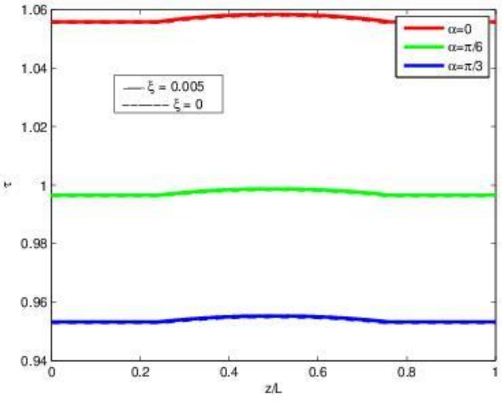


Fig.6 : Variation of wall shear stress **τ** with z/L and for with z/L and for
different values of angle of artery inclination (α).
δ.

In Figure 2 with the increase of magnetic field M the axial velocity shows a reverse behaviour. It is observed that with the increase of magnetic field M, the curve representing the axial flow velocity shift towards the origin. Figure 3 illustrates that variation of axial velocity u with r and the tapered and non tapered angles for different value of Darcy number $\sqrt{D_a}$. It is clear that the axial velocity decreases in increasing value of Darcy number $\sqrt{D_a}$. In Figure 4 shows that increasing the inclination angle of artery as the increases of volumetric flow rate for tapered and non tapered angels. In Figure 5 shows that increasing the value of time t and Darcy number $\sqrt{D_a}$ for volumetric flow rate is also increases. In Figure 6 depicts that the variation of wall shear stress **τ** with z/L for different values of inclination angle of artery (α) for tapered and non tapered angle ξ. It is evident that the wall shear stress decreases with increases of inclination angle of artery(α). It is quite interesting to observe from the figure 7 that as the variation of wall shear stress at the Darcy number $\sqrt{D_a}$. It has been noticed that with the increasing the value of Darcy number as the decreases of wall shear stress.

for different values of $\sqrt{D_a}$ and time t.

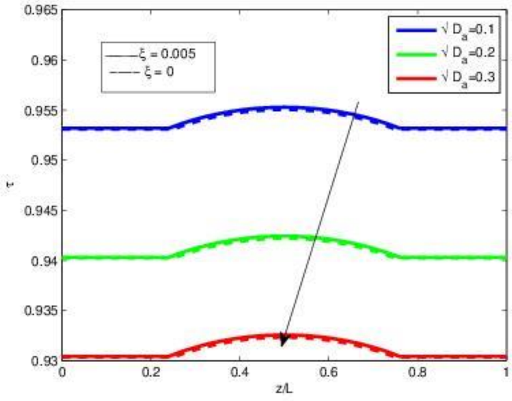


Fig. 7 Variation of wall shear stress **τ**
different values of height of the stenotic region

5. CONCLUSION

In this study we have developed unsteady oscillatory flow of blood in an inclined tapered stenosed artery with permeable wall and Effects of Magnetic field. To observe the effects of wall permeability on blood flow characteristics in a stenosed artery, the flow of a Newtonian fluid through a composite stenosis is a circular tube with permeable wall has been discussed. Analytical expressions of flow variables are obtained and variations of wall shear stress, volumetric flow rate and axial velocity are examined graphically. Since, this study has been carried out for a situation when the human body is subjected to magnetic field. The unsteady is also useful for effects the roll of permeability. This investigation may be helpful for the practioners to treat hypertension patient through magnetic therapy and to understand the flow of blood under stenotic conditions. Finally we can conclude that further potential improvement of the model are anticipated. Since these affects blood pressure, further study should examine the other factors such as diet, tobacco, smoking, overweight etc. from a cardiovascular point of view. Moreover on the basis of the present results, it can be concluded that the flow of blood and pressure can be controlled by the application of an unsteady oscillatory flow.

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